END 294

Yöneylem Araştırması I

Ödev 4 Cevap Anahtarı

Bölüm 4.4 Problem 3. Basic Variables Basic Feasible Solution Corner Point

x_1, x_2	$x_1=150$ $x_2=100$ $s_1=s_2=0$(150, 100)
x_1, s_1	$x_1=200$, $s_1=150$, $x_2=s_2=0$(200, 0)
x_1, s_2	$x_1=350$, $s_2=-300$, $x_2=s_1=0$ Infeasible
x_2, s_1	$x_2=400$, $s_1=-450$, $x_1=s_2=0$ Infeasible
x_2, s_2	$x_2=175$, $s_2 = 225$, $x_1=s_1=0$(0, 175)
s_1, s_2	$s_1=350$, $s_2=400$, $x_1=x_2=0$ (0, 0)

Bölüm 4.12 Problem 4. Adding excess and artificial variables we obtain

min $z = 3x_1 + Ma_1 + Ma_2$

s.t. $2x_1 + x_2 - e_1 + a_1 = 6$

$3x_1 + 2x_2 + a_2 = 4$

Eliminating a_1 and a_2 from $z-3x_1-Ma_1-Ma_2 = 0$ yields

$z + (5M-3)x_1 + 3Mx_2 - Me_1 = 10M$. The simplex now yields

z	x_1	x_2	e_1	a_1	a_2	RHS
1	5M-3	3M	-M	0	0	10M
0	2	1	-1	1	0	6
0	3	2	0	0	1	4

z	x_1	x_2	e_1	a_1	a_2	RHS
1	0	(6-M)/3	-M	0	(3-5M)/3	10M/3 + 4
0	0	-1/3	-1	1	-2/3	10/3
0	1	2/3	0	0	1/3	4/3

This is an optimal tableau. Note, however, that the artificial variable a_1 is positive (a_1 = 10/3) Thus original problem has no feasible solution.

Bölüm 4.14 Problem 2. Let $x_2 = x_2' - x_2''$ where $x_2' >= 0$ and $x_2'' >= 0$. Applying the simplex we obtain

z	x_1	x_2'	x_2''	s_1	s_2	RHS
1	-2	-1	1	0	0	0
0	3	1	-1	1	0	6
0	1	1	-1	0	1	4

z	x_1	x_2'	x_2''	s_1	s_2	RHS
1	0	-1/3	1/3	2/3	0	4
0	1	1/3	-1/3	1/3	0	2
0	0	2/3	-2/3	-1/3	1	2

z	x_1	x_2'	x_2''	s_1	s_2	RHS
1	0	0	0	1/2	1/2	5
0	1	0	0	1/2	-1/2	1
0	0	1	-1	-1/2	3/2	3

This is an optimal tableau with optimal solution z = 5, $x_1 = 1$, $x_2' = 3$, $x_2'' = 0$, $s_1 = s_2 = 0$. Thus the optimal solution has $x_2 = 3-0 = 3$.

Bölüm 4 Review Problem 3.

max $z = 5x_1 - x_2 - Ma_1 = 0$

s.t. $2x_1 + x_2 + a_1 = 6$

$x_1 + x_2 + s_2 = 4$

$x_1 + 2x_2 + s_3 = 5$

After eliminating a_1 from row 0, we obtain $z - (2M + 5)x_1 + (1 - M)x_2 = -6M$.

z	x_1	x_2	a_1	s_2	s_3	RHS
1	-2M-5	1-M	0	0	0	-6M
0	2	1	1	0	0	6
0	1	1	0	1	0	4
0	1	2	0	0	1	5

z	x_1	x_2	a_1	s_2	s_3	RHS
1	0	7/2	(2M+5)/2	0	0	15
0	1	1/2	1/2	0	0	3
0	0	1/2	-1/2	1	0	1
0	0	3/2	-1/2	0	1	2

This is an optimal tableau with the optimal solution being z = 15, $x_1 = 3$, $x_2 = 0$.

Bölüm 4 Review Problem 7.

z	x_1	x_2	s_1	s_2	s_3	RHS	Basic Var.
1	-4	-1	0	0	0	0	z=0
0	2	3	1	0	0	4	s_1=4
0	1	1	0	1	0	1	s_2=1
0	4	1	0	0	1	2	s_3=2

z	x_1	x_2	s_1	s_2	s_3	RHS	Basic Var.
1	0	0	0	0	1	2	z=2
0	0	5/2	1	0	-1/2	3	s_1=3
0	0	3/4	0	1	-1/4	1/2	s_2=1/2
0	1	1/4	0	0	1/4	1/2	x_1=1/2

This tableau yields the optimal solution z = 2, x_1 = 1/2, x_2 = 0. Pivoting in the non-basic variable x_2 yields the alternative optimal solution z = 2, x_1 = 1/3, x_2 = 2/3. Averaging these two optimal solutions yields a third optimal solution: z = 2, x_1 = 5/12, x_2 = 1/3. The LP has an infinite number of optimal solutions.

Bölüm 4 Review Problem 11.

a.

Basic Variables	Basic Feasible Solution
x_1, x_2	x_1 = 1, x_2 = 80 (bfs 1)
x_1, s_1	Infeasible
x_1, s_2	x_1 =1, s_2 = 80 (bfs 2)
x_2, s_1	x_2 = 100, s_1 = 1 (bfs 3)
x_2, s_2	Infeasible
s_1, s_2	s_1 =1, s_2 = 100 (bfs 4)

b.

z	x_1	x_2	s_1	s_2	RHS
1	-10	-1	0	0	0
0	1	0	1	0	1
0	20	1	0	1	100

z	x_1	x_2	s_1	s_2	RHS
1	0	-1	10	0	10
0	1	0	1	0	1
0	0	1	-20	1	80

z	x_1	x_2	s_1	s_2	RHS
1	0	0	-10	1	90
0	1	0	1	0	1
0	0	1	-20	1	80

z	x_1	x_2	s_1	s_2	RHS
1	10	0	0	1	100
0	1	0	1	0	1
0	20	1	0	1	100

This is an optimal tableau. The simplex examined bfs 4, then bfs 2, then bfs 1, before it found the optimal solution (bfs 3) z = 100, $x_1 = 0$, $x_2 = 100$.

Bölüm 4 Review Problem 18.

a. $b \geq 0$ is necessary. If $c_1 = 0$ and $c_2 \geq 0$ we can pivot in x_1 to obtain an alternative optimum. If $c_1 \geq 0$, $c_2 \geq 0$ and $a_2 > 0$ we can pivot in x_5 and obtain an alternative optimum. If $c_2 = 0$,
$a_1 > 0$ and $c_1 \geq 0$ we can pivot in x_2 and obtain an alternative optimum.

b. $b < 0$

c. $b = 0$

d. $b \geq 0$ makes the solution feasible. If $c_2 < 0$ and $a_1 \leq 0$ we can make x_2 as large as desired and obtained an unbounded solution.

e. $b \geq 0$ makes the current basic solution feasible. For x_6 to replace x_1 we need $c_1 < 0$ (this ensures that increasing x_1 will increase z) and we need Row 3 to win the ratio test for x_1.
This requires $3/a_3 \leq b/4$.